

# $Z_b(10650)$ and $Z_b(10610)$ states in a chiral quark model

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We perform a systematic study of  $B\bar{B}^*$ ,  $B^*\bar{B}^*$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  systems by using effective interaction in our chiral quark model. Our results show that the interactions of  $B\bar{B}^*$ ,  $B^*\bar{B}^*$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  states are attractive, which consequently result in  $B\bar{B}^*$ ,  $B^*\bar{B}^*$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  bound states. The recent observed exotic-like hadrons of  $Z_b(10610)$  and  $Z_b(10650)$  are, therefore in our approach, interpreted as loosely bound states of  $B\bar{B}^*$  and  $B^*\bar{B}^*$ , while  $X(3872)$  and  $Y(3940)$  are  $D\bar{D}^*$  and  $D^*\bar{D}^*$  molecule states, respectively.

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## I. INTRODUCTION

Recently, the Belle Collaboration pronounced two charged bottomonium-like resonances  $Z_b(10610)$  and  $Z_b(10650)$  [1]. These two resonances were observed in the five decay channels of  $\Upsilon(5S) \rightarrow \pi^\pm \Upsilon(nS)$  ( $n = 1, 2, 3$ ) and  $\Upsilon(5S) \rightarrow \pi^\pm h_b(mP)$  ( $m = 1, 2$ ). The recommended masses and widths of the two resonances are  $M(10610) = 10608.4 \pm 2.0$  MeV/ $c^2$ ,  $\Gamma_1 = 15.6 \pm 2.5$  MeV/ $c^2$  and  $M(10650) = 10653.2 \pm 1.5$  MeV/ $c^2$ ,  $\Gamma_2 = 14.4 \pm 3.2$  MeV/ $c^2$ , respectively. For both states, experimental results fixed their quantum numbers as  $I^G(J^P)=1^+(1^+)$ . The two resonances can't be simply interpreted as the  $b\bar{b}$  heavy quarkoniums because they are charged. It is suggested that they might be  $B\bar{B}^*$  and  $B^*\bar{B}^*$  molecules, respectively, since their masses are near to the thresholds of  $B\bar{B}^*$  and  $B^*\bar{B}^*$ , and they have narrow widths.

There are many different explanations about the structures of  $Z_b(10610)$  and  $Z_b(10650)$  in the literature [2–15]. Ali et al. [2, 3] and Guo et al. [4] supported that  $Z_b(10610)$  and  $Z_b(10650)$  could be explained as tetraquark states. Cui et al. [5] showed both tetraquark and molecule interpretations were compatible with the experimental measurements in the QCD sum rule calculation. However, Matheus [6] and Chen [7] argued that the masses of tetraquark states were obviously below the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  thresholds. Zhang et al. [8] showed their quite satisfactory result that  $Z_b(10610)$  might be regarded as a  $B\bar{B}^*$  molecule which agreed with the Belle's molecule suggestion [1]. Moreover, Nieves and Valderama argued that there might be three isoscalar  $B\bar{B}^*$  bound states by considering one pion exchange interaction, but all of the three didn't fit the quantum numbers of  $Z_b(10610)$  [9]. Ding et al. [10, 11] suggested  $B\bar{B}^*$  and  $B^*\bar{B}^*$  could form molecule bound states, but these states couldn't fit to the  $1^+(1^+)$  quantum numbers of the two  $Z_b$  states. By including only screened color-Coulomb and screened linear confining potentials, Wong [12] also got some  $B\bar{B}^*$  and  $B^*\bar{B}^*$  molecules, but his  $Z_b(10610)$  and  $Z_b(10650)$  should be the first radial excitation of  $B\bar{B}^*$  and  $B^*\bar{B}^*$  molecules, respectively. In addition, by considering both one boson exchange interaction and the one pion exchange interaction and considering the  $S - D$

mixing between  $B/B^*$  and  $\bar{B}^*$ , Sun et al. [13] gave their result of  $Z_b(10610)$  and  $Z_b(10650)$  in the molecule scenario. It should be mentioned that the calculations of Dong et al. [14] and Cleven et al. [15] supported the molecule scenario, however, the latter couldn't exclude the tetraquark component.

We see that by using different approaches one may have different interpretations for the two new resonances of  $Z_b(10650)$  and  $Z_b(10610)$ . In this work, we will study the two resonances based on our chiral quark model. It should be stressed that the chiral quark model is successful to well describe the binding energies of the baryon ground states, the binding energy of the deuteron, the  $NN$  scattering phase shifts and the  $NY$  cross sections simultaneously [16–25]. It is expected that the chiral quark model provides another way to study the possible  $B\bar{B}^*$  and  $B^*\bar{B}^*$  bound states problem on quark level.

In addition, it is known that the study of the hadronic states of  $X(3872)$  observed in Ref. [26–29] and  $Y(3940)$  observed in Ref. [30–33] have been a hot issue recently. This is because they have exotic properties and can't be simply regarded as  $c\bar{c}$  charmoniums. Among the various approaches, hadronic molecule of  $0^+(1^{++})$   $D\bar{D}^*$  is one feasible interpretation of  $X(3872)$  [34–39]. Therefore, analogous to the systems of  $B\bar{B}^*$  and  $B^*\bar{B}^*$ , it is also of significance to study the  $D\bar{D}^*$  and  $D^*\bar{D}^*$  systems in our chiral quark model for  $X(3872)$  and  $Y(3940)$ .

It should be mentioned that several works based on our chiral quark model have been published [16–18] to study the bound states and binding energies of heavy-meson molecule systems with Resonating Group Method (RGM). The RGM method is a successful one proficiently adopted in Ref. [16–25]. In this work, different from the RGM method, we apply the approach introduced in Ref. [40] to derive the analytical form of the total interaction potentials between the two S-wave heavy mesons. Here the interaction potentials result from the one meson exchange interactions between two light quarks. We anticipate that this method would give a more accurate description of the short-range interaction between the two clusters in the chiral quark model, and we thoroughly investigate the possible bound states of  $B\bar{B}^*$  and  $B^*\bar{B}^*$  systems as well as their charm partners  $D\bar{D}^*$  and  $D^*\bar{D}^*$

by solving the Schrödinger Equation with our analytical interaction potentials between the two clusters.

The paper is organized as follows. In section II, the framework of our chiral quark model is briefly introduced, and the analytical forms of the effective interaction potentials between the two S-wave meson clusters under our chiral quark model are given. The bound state solutions for the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  systems and their charm partners of  $D\bar{D}^*$  and  $D^*\bar{D}^*$  are shown in Sec. III. Finally, the summary is given in Sec. IV.

## II. FORMULATION

The framework of our effective chiral quark model has been discussed extensively in the literature [16, 17, 19–21]. Here we just briefly present the main part of the model. The total Hamiltonian of the approach reads

$$H = T_q - T_G + \sum_{i,j} V(r_{ij}), \quad (1)$$

where  $T_q$  is the sum of the kinetic energy operator of all the quarks and antiquarks in the two clusters, and  $T_G$  is the kinetic energy operator of the center-of-mass motion. In a molecule scenario or cluster model, because the interactions inside each cluster don't dedicate to the total interaction between the two clusters of the system, only the interactions between the two different clusters need to be taken into account.  $V(r_{ij})$  in Eq. 1 represents the interactions between the  $i$ th light quark or antiquark in the first cluster and the  $j$ th light quark or antiquark in another one, and

$$V(r_{ij}) = V^{OGE}(r_{ij}) + V^{conf}(r_{ij}) + V^M(r_{ij}). \quad (2)$$

Here  $V_{ij}^{OGE}$  is the one-gluon exchange (OGE) potential

$$V^{OGE}(r_{ij}) = \frac{1}{4}g_i g_j (\lambda_i^c \cdot \lambda_j^c) \left[ \frac{1}{r_{ij}} - \frac{\pi}{2}\delta(r_{ij}) \right. \\ \left. \times \left( \frac{1}{m_{q_i}^2} + \frac{1}{m_{q_j}^2} + \frac{4}{3} \frac{\sigma_i \cdot \sigma_j}{m_{q_i} m_{q_j}} \right) \right], \quad (3)$$

and  $V_{ij}^{conf}$  is the confinement

$$V^{conf}(r_{ij}) = -(\lambda_i^c \cdot \lambda_j^c) (a_{ij}^c r_{ij} + a_{ij}^{c0}). \quad (4)$$

$V^M(r_{ij})$  in Eq. 2 stands for the total one-meson exchange interaction potential. Generally,

$$V^M(r_{ij}) = \sum_{a=0}^8 V^{\sigma_a}(r_{ij}) + \sum_{a=0}^8 V^{\pi_a}(r_{ij}) + \sum_{a=0}^8 V^{\rho_a}(r_{ij}), \quad (5)$$

with  $V^{\sigma_a}(r_{ij})$ ,  $V^{\pi_a}(r_{ij})$  and  $V^{\rho_a}(r_{ij})$  being the interactions induced from scalar meson, pseudoscalar meson, and vector meson exchanges, respectively.

For light quark-quark interaction,  $V^{\sigma_a}(r_{ij})$ ,  $V^{\pi_a}(r_{ij})$  and  $V^{\rho_a}(r_{ij})$  have such forms:

$$V^{\sigma_a}(r_{ij}) = -C(g_{ch}, m_{\sigma_a}, \Lambda) X_1(m_{\sigma_a}, \Lambda, r_{ij}) (\lambda_i^a \lambda_j^a), \quad (6)$$

$$V^{\pi_a}(r_{ij}) = C(g_{ch}, m_{\pi_a}, \Lambda) \frac{m_{\pi_a}^2}{12m_i m_j} X_2(m_{\pi_a}, \Lambda, r_{ij}) \\ \times (\sigma_i \cdot \sigma_j) (\lambda_i^a \lambda_j^a), \quad (7)$$

$$V^{\rho_a}(r_{ij}) = C(g_{chv}, m_{\rho_a}, \Lambda) \left[ X_1(m_{\rho_a}, \Lambda, r_{ij}) + \frac{m_{\rho_a}^2}{6m_i m_j} \right. \\ \left. \times \left( 1 + \frac{f_{chv}}{g_{chv}} \frac{m_i + m_j}{M_N} + \frac{f_{chv}^2}{g_{chv}^2} \frac{m_i m_j}{M_N^2} \right) \right. \\ \left. \times X_2(m_{\rho_a}, \Lambda, r_{ij}) (\sigma_i \cdot \sigma_j) \right] (\lambda_i^a \lambda_j^a), \quad (8)$$

where  $\lambda^a$  is the Gell-Mann matrix in flavor space. In our chiral SU(3) quark model, we only consider the scalar meson and pseudoscalar meson exchanges, while in our extended chiral SU(3) quark model vector meson exchange interactions are also included.  $m_{\sigma_a}$ ,  $m_{\pi_a}$  and  $m_{\rho_a}$  in Eq. 5 are the masses of the scalar nonets, the pseudoscalar nonets and the vector nonets, respectively.  $M_N$  in Eq. 8 is a mass scale usually taken as the mass of nucleon [21]. In the above equations,  $m_i$  is the  $i$ th light constituent quark mass in the first cluster,  $m_j$  is the  $j$ th light constituent quark mass in the second cluster.  $g_{ch}$  is the coupling constants for the scalar and pseudoscalar nonets.  $g_{chv}$  and  $f_{chv}$  are the coupling constants for the vector coupling and tensor coupling of vector nonets, respectively. In Eqs. 6, 7, 8,

$$C(g_{ch}, m, \Lambda) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m, \quad (9)$$

$$X_1(m, \Lambda, r_{ij}) = Y(mr_{ij}) - \frac{\Lambda}{m} Y(\Lambda r_{ij}), \quad (10)$$

$$X_2(m, \Lambda, r_{ij}) = Y(mr_{ij}) - \left( \frac{\Lambda}{m} \right)^3 Y(\Lambda r_{ij}), \quad (11)$$

$$Y(x) = \frac{1}{x} e^{-x}. \quad (12)$$

For light quark-antiquark interactions,  $G_{\sigma_a, \pi_a, \rho_a}$ , the  $G$ -parity of the exchanged meson, should be included in the interaction potentials  $V^{\sigma_a}(r_{ij})$ ,  $V^{\pi_a}(r_{ij})$  and  $V^{\rho_a}(r_{ij})$ .

In the  $B\bar{B}^*$ ,  $B^*\bar{B}^*$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  two-heavy-meson systems, only the light quark-antiquark interactions are considered. To solve the bound state problem for these systems, unlike the Resonating Group Method (RGM) employed in our previous works [16, 17], we use the approach explicitly discussed in Ref. [40]. We first derive an analytical form of the total effective interaction between the two mesons, and then by solving the Schrödinger Equation, we calculate the binding energy  $E$ , and finally draw a conclusion whether the molecule bound states could exist in the two-meson systems or not.

Now corresponding to scalar, pseudoscalar and vector meson exchanges (see Eqs. 6, 7 and 8), using the method described in Ref. [40], we get the analytical effective interaction potentials between the two S-wave heavy meson clusters as

$$\begin{aligned} V_{q\bar{q}}^{\sigma_a}(\xi) &= -G_{\sigma_a} C(g_{ch}, m_{\sigma_a}, \Lambda) X_{1q\bar{q}}(m_{\sigma_a}, \Lambda, \xi) (\lambda_q^a \lambda_{\bar{q}}^a) 13 \\ V_{q\bar{q}}^{\pi_a}(\xi) &= G_{\pi_a} C(g_{ch}, m_{\pi_a}, \Lambda) \frac{m_{\pi_a}^2}{12m_q m_{\bar{q}}} X_{2q\bar{q}}(m_{\pi_a}, \Lambda, \xi) \\ &\quad \times (\sigma_q \cdot \sigma_{\bar{q}}) (\lambda_q^a \lambda_{\bar{q}}^a), \end{aligned} \quad (14)$$

$$\begin{aligned} V_{q\bar{q}}^{\rho_a}(\xi) &= G_{\rho_a} C(g_{chv}, m_{\rho_a}, \Lambda) \left[ X_{1q\bar{q}}(m_{\rho_a}, \Lambda, \xi) \right. \\ &\quad \left. + \frac{m_{\rho_a}^2}{6m_q m_{\bar{q}}} \left( 1 + \frac{f_{chv}}{g_{chv}} \frac{m_q + m_{\bar{q}}}{M_N} + \frac{f_{chv}^2}{g_{chv}^2} \frac{m_q m_{\bar{q}}}{M_N^2} \right) \right. \\ &\quad \left. \times X_{2q\bar{q}}(m_{\rho_a}, \Lambda, \xi) (\sigma_q \cdot \sigma_{\bar{q}}) \right] (\lambda_q^a \lambda_{\bar{q}}^a). \end{aligned} \quad (15)$$

Here,  $\xi$  is the relative coordinate between two different clusters, namely, the relative coordinate between the two centers-of-mass of the two clusters, and

$$X_{1q\bar{q}}(m, \Lambda, \xi) = Y_{q\bar{q}}(m\xi) - \frac{\Lambda}{m} Y_{q\bar{q}}(\Lambda\xi), \quad (16)$$

$$X_{2q\bar{q}}(m, \Lambda, \xi) = Y_{q\bar{q}}(m\xi) - \left( \frac{\Lambda}{m} \right)^3 Y_{q\bar{q}}(\Lambda\xi). \quad (17)$$

The modified Yukawa term reads

$$\begin{aligned} Y_{q\bar{q}}(m\xi) &= \frac{1}{2m\xi} e^{\frac{m^2}{4\beta}} \left\{ e^{-m\xi} \left\{ 1 - \text{erf} \left[ -\sqrt{\beta} \left( \xi - \frac{m}{2\beta} \right) \right] \right\} \right. \\ &\quad \left. - e^{m\xi} \left\{ 1 - \text{erf} \left[ \sqrt{\beta} \left( \xi + \frac{m}{2\beta} \right) \right] \right\} \right\}. \end{aligned} \quad (18)$$

Here,

$$\beta = \frac{\mu_{q\bar{Q}} \mu_{Q\bar{q}} \omega}{\mu_{q\bar{Q}} \left( \frac{m_Q}{m_Q + m_{\bar{q}}} \right)^2 + \mu_{Q\bar{q}} \left( \frac{m_{\bar{Q}}}{m_q + m_{\bar{Q}}} \right)^2}, \quad (19)$$

and  $\omega$  is the harmonic-oscillator frequency of the light quark wave function.  $q$  ( $\bar{q}$ ) and  $Q$  ( $\bar{Q}$ ) in Eq. 19 are the light quark (antiquark) and heavy quark (antiquark) in the two clusters, respectively.

There are some necessary parameters in the interaction potentials in our chiral quark model. In this work, we adopt the parameters determined in our previous works [16, 17, 19–24]. The up/down quark mass  $m_q$  is taken as 313 MeV. The harmonic-oscillator frequency  $\omega$  is  $2.522 fm^{-1}$  in the chiral SU(3) quark model and  $3.113 fm^{-1}$  in the extended chiral SU(3) quark model. The coupling constant for the scalar and pseudoscalar chiral field  $g_{ch} = 2.621$  is fixed by the relation of

$$\frac{g_{ch}^2}{4\pi} = \frac{9}{25} \frac{g_{NN\pi}^2}{4\pi} \frac{m_u^2}{M_N^2},$$

with  $g_{NN\pi}^2/4\pi = 13.67$  determined from experiments. In our extended chiral SU(3) quark model, vector meson exchange interactions are included. The coupling constant for vector coupling  $g_{chv}$  in Eq. 8 is taken to be 2.351 and 1.973, respectively. Correspondingly, the ratio of the tensor coupling  $f_{chv}$  to the vector coupling  $g_{chv}$  is taken to be 0 and 2/3, respectively. In our calculation, the masses of the mesons are taken from the PDG [41], except the  $\sigma$  meson, which does not have a well-defined value. Here  $m_\sigma$  is obtained by fitting the binding energy of the deuteron [21]. It is  $m_\sigma = 595$  MeV in our chiral SU(3) quark model, 535 MeV and 547 MeV for  $f_{chv}/g_{chv}$  respectively taken as 0 and 2/3 in the extended chiral SU(3) quark model. The cutoff  $\Lambda$  is taken as 1100 MeV. Since there is no color-interrelated interaction between the two color-singlet clusters, we don't have to consider the OGE and confinement interactions, and therefore, we don't list the OGE coupling constant  $g_{i,j}$  in Eq. 3 and the parameters of the confinement potential  $a_{ij}^c$  and  $a_{ij}^0$  in Eq. 4.

The remaining parameters to be determined are the heavy quark masses  $m_c$  and  $m_b$ . In our work, we find that the final results are not sensitive to the variation of the heavy quark masses, and we take  $m_c = 1430$  MeV [42] and  $m_b = 4720$  MeV [43] as typical values.

### III. NUMERICAL SOLUTIONS

To make sure the  $B\bar{B}^*$ ,  $B^*\bar{B}^*$  systems and their charm partners of  $D\bar{D}^*$ ,  $D^*\bar{D}^*$  have definite quantum numbers of isospin  $I$  and  $C$ -parity  $C$ , we construct the following flavor wave functions as Refs. [13, 16, 39]. For  $B\bar{B}^*$  system:

$$I = 1 : \begin{cases} \frac{1}{\sqrt{2}} (B^{*+} \bar{B}^0 + c B^+ \bar{B}^{*0}) \\ \frac{1}{\sqrt{2}} (B^{*-} \bar{B}^0 + c B^- \bar{B}^{*0}) \\ \frac{1}{2} [(B^{*0} \bar{B}^0 - B^{*+} B^-) + c (B^0 \bar{B}^{*0} - B^+ B^{*-})], \end{cases} \quad (20)$$

$$I = 0 : \frac{1}{2} [(B^{*0} \bar{B}^0 + B^{*+} B^-) + c (B^0 \bar{B}^{*0} + B^+ B^{*-})]. \quad (21)$$

Here,  $c=1$  for  $C=+$  and  $c=-1$  for  $C=-$ . For  $B^*\bar{B}^*$  system, the flavor function is

$$I = 1 : \begin{cases} B^{*+} \bar{B}^{*0} \\ B^{*-} \bar{B}^{*0} \\ \frac{1}{\sqrt{2}} (B^{*0} \bar{B}^{*0} - B^{*+} B^{*-}), \end{cases} \quad (22)$$

$$I = 0 : \frac{1}{\sqrt{2}} (B^{*0} \bar{B}^{*0} + B^{*+} B^{*-}). \quad (23)$$

For  $D\bar{D}^*$  and  $D^*\bar{D}^*$  systems, the similar expressions can be obtained.

### A. $B\bar{B}^*$

By using the analytical effective interactions we have gotten in section II, we study the  $B\bar{B}^*$  system with different isospin  $I$  and  $C$ -parity  $C$ . After solving the Schrödinger Equation with the programs developed in Refs. [44, 45] and with the interactions in Eq. 13,14,15, we list the obtained results in Table I. In the table, I, II and III refer to the chiral SU(3) quark model, the extended chiral SU(3) quark models with vector nonets coupling constant  $f_{chv}/g_{chv}=0$  and  $2/3$ , respectively. If bound state does exist, we list, in the table, the binding energy  $E$  in MeV. The symbol "—" denotes that bound state doesn't exist.

TABLE I: Binding energies (MeV) of the bound states of  $B\bar{B}^*$  system with different  $I$  and  $C$ .

$I$	$C$	I	II	III
0	+	46.74	97.75	78.04
	—	3.27	66.53	50.59
1	+	—	—	—
	—	0.39	1.75	1.49

From Table I, we see there are three  $S$ -wave  $B\bar{B}^*$  bound states with quantum numbers  $I^G(J^{PC}) = 0^+(1^{++})$ ,  $0^-(1^{+-})$  and  $1^+(1^{+-})$  respectively. The binding energies of  $0^+(1^{++})$ ,  $0^-(1^{+-})$  and  $1^+(1^{+-})$  bound states are respectively 46.76–97.75 MeV, 3.27–66.53 MeV and 0.39–1.75 MeV in our models. Table I shows that the attractive interaction of  $I=0$   $B\bar{B}^*$  system is much stronger than that of  $I=1$   $B\bar{B}^*$  system and the attractive interaction in the extended chiral SU(3) quark model (models II and III) is stronger than that in the chiral SU(3) quark model (model I). From our calculation, we notice that in  $B\bar{B}^*$  system,  $\sigma$ ,  $\sigma'$ ,  $\pi$ ,  $\omega$ , and  $\rho$  exchange interactions play a dominant role and they determine whether  $B\bar{B}^*$  bound state exists or not.

For the  $I=0$   $C=+$  case, in the chiral SU(3) quark model,  $\sigma$ ,  $\sigma'$  and  $\pi$  exchanges provide strong attraction, so the total interaction is strong enough to form a  $B\bar{B}^*$  bound state. In the extended chiral SU(3) quark model, the contributions of vector meson exchange are also included, and  $\rho$  and  $\omega$  exchanges provide additional attraction. Therefore, the  $B\bar{B}^*$  system has a larger binding energy as shown in Table I where this  $0^+(1^{++})$  bound state has a binding energy 46.76–97.75 MeV.

For the  $I=0$   $C=-$  case, in the chiral SU(3) quark model,  $\sigma$  and  $\sigma'$  exchanges provide strong attraction but  $\pi$  exchange provides strong repulsion, as a result of the competition, the  $B\bar{B}^*$  still binds but the binding energy becomes much smaller. In the extended chiral SU(3) quark model,  $\rho$  and  $\omega$  exchanges provide very strong attraction as well, so the binding energy becomes larger than the one in the chiral quark model. This  $0^-(1^{+-})$  bound state has a binding energy 3.27–66.53 MeV.

For the  $I=1$   $C=+$  case, in all the three quark models,  $\sigma$  exchange provides strong attraction but  $\sigma'$  and  $\pi$

exchanges provide strong repulsion, thus the total interaction is weakly attractive, however the  $B\bar{B}^*$  can't bind in our approach.

For the  $I=1$   $C=+$  case, in the chiral SU(3) quark model,  $\sigma$  exchange provides strong attraction but  $\sigma'$  and  $\pi$  exchanges provide strong repulsion, thus the total interaction is weakly attractive, however the  $B\bar{B}^*$  can't bind in our approach. In the extended chiral SU(3) quark model, the strong attraction provided by  $\omega$  exchange and the strong repulsion provided by  $\rho$  exchange almost cancel each other, so the  $B\bar{B}^*$  still can't bind.

For the  $I=1$   $C=-$  case, in the chiral SU(3) quark model,  $\sigma$  and  $\pi$  exchanges provide strong attraction but  $\sigma'$  exchange provides repulsion, so the total interaction is attractive, and the  $B\bar{B}^*$  can form a weakly bound state. In the extended chiral SU(3) quark model, the contributions of vector meson exchange are also cancelled, so the  $B\bar{B}^*$  still can form a  $1^+(1^{+-})$  weakly bound state with weak binding energy 0.39–1.75 MeV. In Fig. 1, the total interaction potential of the charged  $1^+(1^{+-})$   $S$ -wave  $B\bar{B}^*$  system is depicted. This charged  $1^+(1^{+-})$   $S$ -wave  $B\bar{B}^*$  weakly bound state has a mass 10603–10604 MeV, and it might be explained as one possible candidate of  $Z_b(10610)$ .

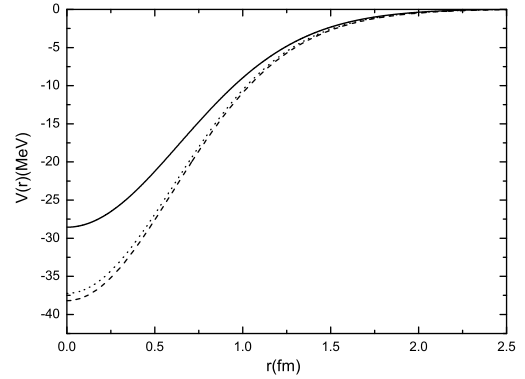


FIG. 1: The total interaction potential of  $I=1$  and  $C=-$   $B\bar{B}^*$  system. The solid, dashed and dotted lines represent the result obtained from the chiral SU(3) quark model, the extended chiral SU(3) quark model with  $f_{chv}/g_{chv}$  taken as 0 and  $2/3$ , respectively.

### B. $B^*\bar{B}^*$

By solving the Schrödinger Equation with the analytical potentials of  $B^*\bar{B}^*$  system with different isospin  $I$  and spin  $S$ , we list our results of the bound state problems of the system in Table II.

According to Table II, we see that four  $S$ -wave bound states with different quantum numbers could exist in  $B^*\bar{B}^*$  system, namely, the bound states of  $1^+(1^{+-})$ ,

TABLE II: Binding energies (MeV) of the bound states of  $B^*\bar{B}^*$  system with different  $I$  and  $S$  quantum numbers. Symbols have the same meaning as in Table I.

$I$	$S$	I	II	III
0	0	—	51.65	37.55
	1	3.30	66.73	50.77
	2	46.91	97.98	78.24
1	0	1.51	3.66	3.32
	1	0.41	1.78	1.52
	2	—	—	—

with binding energy 0.41–1.78 MeV,  $0^-(1^{+-})$  with binding energy 3.30–66.73 MeV,  $0^+(2^{++})$  with binding energy 46.91–97.98 MeV and  $1^-(0^{++})$  with binding energy 1.51–3.66 MeV. From our calculation, we notice that, in  $B^*\bar{B}^*$  system,  $\sigma$ ,  $\sigma'$ ,  $\pi$ ,  $\omega$  and  $\rho$  exchanges also play a dominant role to determine whether  $B^*\bar{B}^*$  could form a bound state or not.

For the  $I=0$   $S=0$  case, in the chiral SU(3) quark model (model I),  $\sigma$  and  $\sigma'$  exchanges provide attraction, however,  $\pi$  exchange provides strong repulsion. As a result of this competition, the total interaction is too weak to bind  $B^*\bar{B}^*$ . In the extended chiral SU(3) quark model (models II and III), because the contributions of vector meson exchange are also included, and  $\rho$  and  $\omega$  exchanges also provide very strong attraction, the  $B^*\bar{B}^*$  can form a bound state.

For the  $I=0$   $S=1$  case, in the chiral SU(3) quark model,  $\sigma$  and  $\sigma'$  exchanges provide attraction but  $\pi$  exchange provides repulsion, and the total interaction in the  $B^*\bar{B}^*$  system is still strong enough to form a bound state. In the extended chiral SU(3) quark model, the additional  $\rho$  and  $\omega$  exchanges provide very strong attraction, therefore, the binding energy becomes larger. This  $0^-(1^{+-})$  bound state has a binding energy of 3.30–66.73 MeV.

For the  $I=0$   $S=2$  case, in the chiral SU(3) quark model, all  $\sigma$ ,  $\sigma'$  and  $\pi$  exchanges provide strong attraction, so the  $B^*\bar{B}^*$  forms a bound state. Moreover, in the extended chiral SU(3) quark model,  $\rho$  and  $\omega$  exchanges provide additional attraction, so the  $B^*\bar{B}^*$  system could be deeply bound. This  $0^+(2^{++})$  bound state has a binding energy 46.91–97.98 MeV.

For the two cases of  $I=1$   $S=0$  and 1, in the chiral SU(3) quark model,  $\sigma$  and  $\pi$  exchanges provide attraction, and  $\sigma'$  exchange provides repulsion. However, the total interaction is attractive, and the  $B^*\bar{B}^*$  can form a weakly bound state. In the extended chiral SU(3) quark model, like the  $I=1$   $B\bar{B}^*$  system, the contributions of vector meson exchange are also almost cancelled, and the  $B^*\bar{B}^*$  still can form weakly bound states. The  $1^-(0^{++})$  and  $1^+(1^{+-})$  bound states have weakly binding energies 1.51–3.66 MeV and 0.41–1.78 MeV, respectively. In Fig. 2, the total interaction potential of the charged  $1^+(1^+)$   $S$ -wave  $B^*\bar{B}^*$  system is plotted. It should be stressed that this  $1^+(1^+)$   $B^*\bar{B}^*$  weakly bound state has a mass 10648–10650 MeV, which is consistent with the mass and

quantum numbers of  $Z_b(10650)$ . As a result, the new resonance of  $Z_b(10650)$  might be explained as the weakly bound state of  $1^+(1^+)$   $B^*\bar{B}^*$  in our approach.

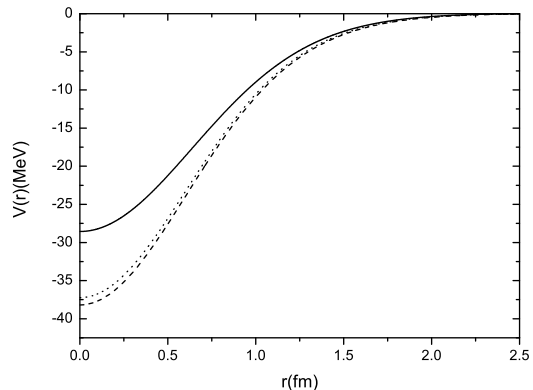


FIG. 2: The total interaction potential of  $I=1$   $S=1$   $B^*\bar{B}^*$  system. Symbols have the same meaning as in Fig. 1.

Comparing Fig. 2 with Fig. 1, we see the two interaction potentials are almost the same. It is not surprising because in these two cases, the spin matrix elements in Eqs. 14 and 15 are both  $-1$  and the other parts of the interaction potentials are the same as each other. The  $B^*\bar{B}^*$  system has a slightly larger binding energy because of its larger reduced mass than the  $B\bar{B}^*$  system which is introduced when solving the Schrodinger Equation.

For the  $I=1$   $S=2$  case of  $B^*\bar{B}^*$  system, we found, in all the three SU(3) quark models,  $\sigma$  exchange provides attraction but  $\sigma'$  and  $\pi$  exchanges provide repulsion, and the total attractive interaction is too weak to bind the  $B^*\bar{B}^*$ .

### C. $D\bar{D}^*$

The only difference between the systems of  $D\bar{D}^*$  and  $B\bar{B}^*$  is the flavor of heavy quark. Considering the resemblance between  $D\bar{D}^*$  and  $B\bar{B}^*$ , the one meson exchange interaction potentials should have similar properties. In our calculation we find the one meson exchange interaction potentials of  $D\bar{D}^*$  system are a bit larger than those of  $B\bar{B}^*$  system with the same quantum numbers of isospin  $I$  and  $C$ -parity  $C$ , and the total interaction also is a bit larger. That is to say, the heavy quark mass has only a little effect on the interaction potentials (see Eq. 19). After solving the Schrödinger Equation, we list our results in Table III.

From Table III, we find the result of  $D\bar{D}^*$  system is quite different from that of  $B\bar{B}^*$  system. This is because the much lighter reduced mass makes the  $D\bar{D}^*$  system more difficult to form a bound state. We see that only one  $S$ -wave  $0^+(1^{++})$   $D\bar{D}^*$  bound state with binding energy of 12.72–47.39 MeV could exist. Unlike the  $B\bar{B}^*$  system,

TABLE III: Binding energies (MeV) of the bound states of  $D\bar{D}^*$  system with different  $I$  and  $C$  quantum numbers. Symbols have the same meaning as in Table I.

$I$	$C$	I	II	III
0	+	12.72	47.39	33.32
	—	—	24.26	14.24
1	+	—	—	—
	—	—	—	—

the total attractive interactions are too weak to form  $0^-(1^{+-})$  and  $1^+(1^{+-})$   $D\bar{D}^*$  states.

In Fig. 3, the total interaction of  $I=0$   $C=+$   $D\bar{D}^*$  system is shown. Our  $0^+(1^{++})$   $D\bar{D}^*$  bound state has a mass 3832-3867 MeV, and it fits well with the mass of new resonance of  $X(3872)$  observed by [26–29]. Particularly, the quantum numbers of  $0^+(1^{++})$  seem to be more preferable in the recent experiment [46] of  $X(3872)$ . Here we speculate that the study of the  $0^+(1^{++})$   $D\bar{D}^*$  molecule bound state may shed light on the  $X(3872)$  structure.

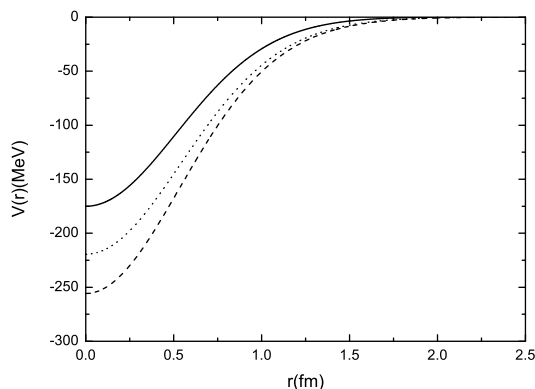


FIG. 3: The total interaction potential of  $I=0$   $C=+$   $D\bar{D}^*$  system. Symbols have the same meaning as in Fig. 1.

#### D. $D^*\bar{D}^*$

In  $D^*\bar{D}^*$  system the one meson exchange interactions have similar properties to those of  $B^*\bar{B}^*$  system. Analogous to the process of  $B^*\bar{B}^*$  system, we list the result of  $D^*\bar{D}^*$  system in Table IV.

Comparing Table IV with Table II, we find the  $D^*\bar{D}^*$  system is more difficult to form bound states than the  $B^*\bar{B}^*$  system. This is because the reduced mass of the  $D^*\bar{D}^*$  system is 1003 MeV which is lighter than that of  $B^*\bar{B}^*$  system of 2663 MeV. For  $D^*\bar{D}^*$  system, we see there is only one  $0^+(2^{++})$   $D^*\bar{D}^*$  bound state with binding energy of 14.33–50.22 MeV. Unlike the  $B^*\bar{B}^*$  system, the total attractive interactions are too weak to form  $0^-(1^{+-})$ ,  $1^-(0^{++})$  and  $1^+(1^{+-})$   $D^*\bar{D}^*$  states.

TABLE IV: Binding energies (MeV) of the bound states of  $D^*\bar{D}^*$  system with different isospin  $I$  and spin  $S$ . Symbols have the same meaning as in Table I.

$I$	$S$	I	II	III
0	0	—	16.15	7.97
	1	—	26.38	15.89
	2	14.33	50.22	35.69
1	0	—	—	—
	1	—	—	—
	2	—	—	—

The total interaction potential of  $I=0$   $S=2$   $D^*\bar{D}^*$  system is depicted in Fig. 4. This  $0^+(2^{++})$   $D^*\bar{D}^*$  state

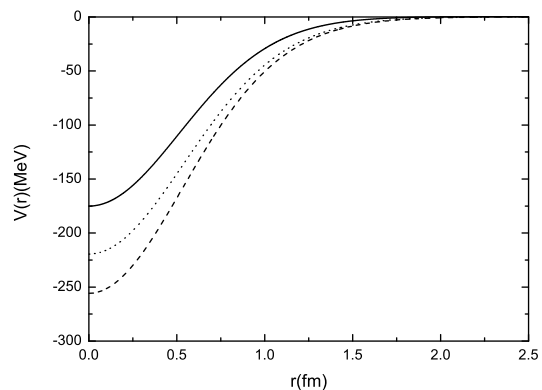


FIG. 4: The total interaction potential of  $I=0$   $S=2$   $D^*\bar{D}^*$  system. Symbols have the same meaning as in Fig. 1.

has a mass of 3964-3999 MeV. We notice that the mass and quantum numbers of this state are consistent with those of  $Y(3940)$  observed recently by Refs. [30–33] and discussed by Refs. [47, 48]. Therefore, the study of our  $0^+(2^{++})$   $D^*\bar{D}^*$  molecule state might be helpful to understand the structure of the  $Y(3940)$ .

#### IV. SUMMARY

In this work, we have performed a systematic investigation of the bound state problem of  $D\bar{D}^*$ ,  $D^*\bar{D}^*$ ,  $B\bar{B}^*$  and  $B^*\bar{B}^*$  systems. In  $D\bar{D}^*$  system, we find only one  $0^+(1^{++})$  bound state and this molecule state might partly account for the structure of  $X(3872)$ . In  $D^*\bar{D}^*$  system, we find only one  $0^+(2^{++})$  bound state could exist and this  $D^*\bar{D}^*$  molecule state might be explained as  $Y(3940)$  observed in experiments.

General speaking, we found that  $B\bar{B}^*$  and  $B^*\bar{B}^*$  can form bound states more easily than  $D\bar{D}^*$  and  $D^*\bar{D}^*$ , because of their much heavier reduced masses. For the two charged new resonances of  $Z_b(10610)$  and  $Z_b(10650)$ , our calculation shows that they might be explained as  $1^+(1^+)$

weakly bound states of  $B\bar{B}^*$  and  $B^*\bar{B}^*$ , respectively. For  $B\bar{B}^*$  system, other two bound states are found in our approach as  $0^+(1^{++})$  with a mass 10506–10557 MeV and  $0^-(1^{+-})$  with a mass 10538–10600 MeV. For  $B^*\bar{B}^*$  system, other three bound states are also found. They are  $0^+(2^{++})$  with a mass 10552–10603 MeV,  $0^-(1^{+-})$  with a mass 10583–10645 MeV, and  $1^+(0^{+-})$  with a mass 10646–10649 MeV. However, until now no experimental evidence has been reported for these five bound states in our approach.

What's more, there are some states unbound in our chiral SU(3) quark model (model I) but bound in our extended chiral SU(3) quark model (model II and model III). So far, we can't draw definite conclusions for them.

It is expected future experimental measurements will provide more information about these new hadron structures and provide a discrimination for our model calculations.

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- [1] I. Adachi et al. (Belle Collaboration), [arXiv:1105.4583 [hep-ex]].
  - [2] A. Ali, C. Hambrock and W. Wang, [arXiv:1110.1333 [hep-ph]].
  - [3] A. Ali, PoS. BEAUTY2011, 002 (2011).
  - [4] T. Guo, L. Cao, M.Z. Zhou and H. Chen, [arXiv:1106.2284 [hep-ph]].
  - [5] C.Y. Cui, Y.L. Liu and M.Q. Huang, [arXiv:1107.1343 [hep-ph]].
  - [6] R.D. Matheus, S. Narison, M. Nielsen, J.M. Richard, Phys. Rev. D **75**, 014005 (2007).
  - [7] W. Chen and S.L. Zhu, Phys. Rev. D **83**, 034010 (2011).
  - [8] J.R. Zhang, M. Zhong and M.Q. Huang, Phys. Lett. B **704**, 312-315 (2011).
  - [9] J. Nieves and M.P. Valderrama, Phys. Rev. D **84**, 056015 (2011).
  - [10] G.J. Ding, Phys. Rev. D **80**, 034005 (2009).
  - [11] G.J. Ding, J.F. Liu and M.L. Yan, Phys. Rev. D **79**, 054005 (2009).
  - [12] C.Y. Wong, Phys. Rev. C **69**, 055205 (2004).
  - [13] Z.F. Sun, J. He, X. Liu, Z.G. Luo and S.L. Zhu, Phys. Rev. D **84**, 054002 (2011).
  - [14] Y.B. Dong, A. Faessler, T. Gutsche, and V.E. Lyubovitskij, "Decays of  $Z_b(+)$  and  $Z_b'(+)$  as hadronic molecules", arXiv: 1203.1894 [hep-ph].
  - [15] M. Cleven, F.K. Guo, C. Hanhart, and U. Meißner, Eur. Phys. J. A **47** 120 (2011).
  - [16] Y.R. Liu and Z.Y. Zhang, Phys. Rev. C **80**, 015208 (2009).
  - [17] Y.R. Liu and Z.Y. Zhang, Phys. Rev. C **79**, 035206 (2009).
  - [18] W.L. Wang, F. Huang, Z.Y. Zhang, and B.S. Zou, Phys. Rev. C **84**, 015203 (2011).
  - [19] Z.Y. Zhang, A. Faessler, U. Straub, L.Ya. Glozman, Nucl. Phys. A **578**, 573 (1994).
  - [20] Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, and U. Straub, Nucl. Phys. A **625**, 59 (1997).
  - [21] L.R. Dai, Z.Y. Zhang, Y.W. Yu, P. Wang, Nucl. Phys. A **727**, 321 (2003).
  - [22] W.L. Wang, F. Huang, Z.Y. Zhang, Y.W. Yu and F. Liu, Eur. Phys. J. A **32**, 293-297 (2007).
  - [23] W.L. Wang, F. Huang, Z.Y. Zhang and F. Liu, J. Phys. G **35**, 085003 (2008).
  - [24] W.L. Wang, F. Huang, Z.Y. Zhang and F. Liu, Mod. Phys. Lett. A **25**, 1325-1332 (2010).
  - [25] W.L. Wang and Z.Y. Zhang, Phys. Rev. C **84**, 054006 (2011).
  - [26] S.-K. Choi et al. (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
  - [27] D. Acosta et al. (CDF II Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
  - [28] V.M. Abazov et al. (DØ Collaboration), Phys. Rev. Lett. **91**, 162002 (2004).
  - [29] B. Aubert et al. (BarBar Collaboration), Phys. Rev. D **71**, 071103 (2005).
  - [30] S.-K. Cho et al. (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).
  - [31] S. Uehara et al. (Belle Collaboration), Phys. Rev. Lett. **96**, 082003 (2006).
  - [32] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **98**, 082001 (2007).
  - [33] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. **101**, 082001 (2008).
  - [34] E.S. Swanson, Phys. Lett. B **598**, 197-202 (2004).
  - [35] E.S. Swanson, Phys Rep **429**, 243-305 (2006).
  - [36] Y.B. Dong, A. Faessler, T. Gutsche, and V.E. Lyubovitskij, Phys. Rev. D **77**, 035205 (2008); Y.B. Dong, Amand Faessler, T. Gutsche, S. Kovalenko, and V.E. Lyubovitskij, Phys. Rev. D **79**, 094013 (2009); Y.B. Dong, A. Faessler, T. Gutsche, and V.E. Lyubovitskij, J. Phys. G **38**, 015001 (2011).
  - [37] I.W. Lee, A. Faessler, T. Gutsche and V.E. Lyubovitskij, Phys. Rev. D **80**, 094005 (2009).
  - [38] Y.R. Liu, X. Liu, W.Z. Deng, S.L. Zhu, Eur. Phys. J. C **56**, 63C73 (2008).
  - [39] X. Liu, Z. G. Luo, Y.R. Liu and S.L. Zhu, Eur. Phys. J. C **61**, 411 (2009).
  - [40] M.T. Li, Y.B. Dong, Z.Y. Zhang, Chin. Phys. C **35** 622-628 (2011), arxiv: 1010.2283[hep-ph]
  - [41] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).
  - [42] H.X. Zhang, W.L. Wang, Y.B. Dai, and Z.Y. Zhang, Commun. Theor. Phys. **49**, 414 (2008).
  - [43] H.X. Zhang, M. Zhang, and Z.Y. Zhang, Chin. Phys. Lett. **24**, 2533 (2007); M. Zhang, H.X. Zhang, and Z.Y. Zhang, Commun. Theor. Phys. **50**, 437 (2008).
  - [44] P. Falkensteiner, H. Grosse, F. Schöberl and P. Hertel, Comput. Phys. Commun. **34**, 287-293 (1985).

- [45] W. Lucha and F. F. Schöberl, Int. J. Mod. Phys. C **10**, 607-619 (1999).
- [46] J.S. Lange, arXiv:1109.1699[hep-ph].
- [47] R. Molina, T. Branz, and E. Oset, Phys. Rev. D **82**, 014010 (2010).
- [48] S. Godfrey, S.L. Olsen, Ann.Rev.Nucl.Part.Sci. **58** 51-73 (2008).